Answers to Coursebook exercises

9 Expressions and formulae

Exercise 9.1 Simplifying algebraic expressions

1 **a** x^9 **b** y^6 **c** z^{10} **d** m^{14} **e** n^{12} **f** p^7
g q^5 **h** r^3 **i** t^5 **j** u^2 **k** v **l** w^7

2 **a** $6x^5$ **b** $12y^9$ **c** $30z^7$ **d** $4m^7$ **e** $4n^{13}$ **f** $8p^3$
g $3q^4$ **h** $3r^4$ **i** $3t^4$ **j** $2u^5$ **k** $2v^4$ **l** $5w$

3 **a** **D** **b** **B** **c** **C** **d** **D**

4 **a** Group 1: all have an x^9 when simplified; $8x^6 \times x^3$, $4x^5 \times 2x^4$, $12x^{10} \div 2x$
Group 2: all have an x^6 when simplified; $6x^3 \times 2x^3$, $12x^8 \div x^2$, $2x^3 \times 3x^3$

b $3x^2 \times 4x^3$. This card doesn't fit as it has an x^5 when simplified.

Exercise 9.2 Constructing algebraic expressions

1 **a** $7n$ **b** $n + 12$ **c** $n - 2$ **d** $20 - n$
e $2n + 9$ **f** $\frac{n}{2}$ **g** $\frac{n}{6} - 4$ **h** n^2
i $\frac{100}{n}$ **j** $2n - 1$ **k** $5(n + 2)$ **l** $8(n - 7)$

2 **a** **i** $2x + 2y$ **ii** xy
b **i** $8x + 6y$ **ii** $12xy$
c **i** $4x$ **ii** x^2
d **i** $8y$ **ii** $4y^2$

3 **a** $P = 2x + 10$, $A = 3x + 6$ **b** $P = 2y - 4$, $A = 4y - 24$
c $P = 4n + 8$, $A = n^2 + 4n$ **d** $P = 10p + 6$, $A = 4p^2 + 12p$

4 **a** **i** 2 red + 2 yellow = 4 green; both = $8x + 4$
ii 3 red + 3 yellow = 6 green; both = $12x + 6$
iii 4 red + 4 yellow = 8 green; both = $16x + 8$
b n red + n yellow = $2n$ green (or similar explanation given in words)
c **i** 3 red + 1 yellow = 6 blue; both = $6x + 6$
ii 6 red + 2 yellow = 12 blue; both = $12x + 12$
iii 9 red + 3 yellow = 18 blue; both = $18x + 18$
d $3n$ red + n yellow = $6n$ blue (or similar explanation given in words)

Exercise 9.3 Substituting into expressions

1 **a** 9 **b** 4 **c** 9 **d** 2
e 8 **f** 0 **g** 8 **h** 30
i 5 **j** 47 **k** –30 **l** –4

2 **a** 21 **b** 36 **c** 10 **d** 16
e 68 **f** 64 **g** 3 **h** –18
i 18 **j** –25 **k** –7 **l** 5

3 **a** For example: Let $x = 2$; $3x^2 = 3 \times 2^2 = 12$ and $(3x)^2 = (3 \times 2)^2 = 36$, so $3x^2 \neq (3x)^2$
b For example: Let $y = 4$; $(-y)^2 = (-4)^2 = 16$ and $-y^2 = -(4^2) = -16$, so $(-y)^2 \neq -y^2$
c For example: Let $a = 2$ and $b = 3$; $2(a + b) = 3(2 + 3) = 15$ and $2a + b = 2 \times 2 + 3 = 7$, so $2(a + b) \neq 2a + b$

Unit 9 Answers to Coursebook exercises

Exercise 9.4 Deriving and using formulae

1 **a** $S = 60M$ **b** $S = 900$ **c** $M = \frac{S}{60}$ **d** $M = 22.5$

2 **a** $F = 60$ **b** $F = -78$ **c** $m = 12$ **d** $a = -1.75$

3 **a** $v = 87$ **b** $v = 125$ **c** $u = 27$ **d** $u = 46$
e $t = 10$ **f** $a = 2$

4 **a** $x + 2$ **b** $T = 2x + 2$ **c** $T = 40$ **d** $x = \frac{T-2}{2}$ **e** $x = 23$

5 **a** 20% **b** 60% **c** 125%

6 **a** 65 kg **b** 49.1 kg (1 d.p.) **c** 95.9 kg (1 d.p.) **d** 57.3 kg (1 d.p.)

7 Sasha is correct as 30 °C = 86 °F and 86 °F > 82 °F (or 82 °F = 27.8 °C and 27.8 °C < 30 °C).

8 **a** She is not underweight as her BMI is 20.05, which is greater than 18.5.
b 3.7 kg

Exercise 9.5 Factorising

1 **a** $3(x + 2)$ **b** $5(2y - 3)$ **c** $6y(x + 2)$ **d** $x(4x + 1)$ **e** $3(3 - 4y)$ **f** $y(2y - 7)$

2 **a** $2(x + 2)$ **b** $3(y - 6)$ **c** $5(2z + 1)$ **d** $4(2a - 1)$ **e** $2(2b + 3)$ **f** $4(4n - 5)$
g $5(2 - x)$ **h** $7(2 + 3x)$ **i** $2(4 - 5y)$ **j** $6(3 + 4z)$ **k** $3(3 + 5m)$ **l** $10(3 - 2k)$

3 **a** $x(3x + 1)$ **b** $6y(y - 2)$ **c** $z(z + 4)$ **d** $2a(2 - a)$ **e** $3b(1 + 3b)$ **f** $3n(4 - 5n)$
g $9(2y - x)$ **h** $3(4y + 3x)$ **i** $4y(2x - 1)$ **j** $5z(3 + 2y)$ **k** $2m(7 + 3n)$ **l** $13k(2 - p)$

4 **a** $2(x + 3y + 4)$ **b** $4(y - 2 + x)$ **c** $3(3xy + 4y - 5)$
d $x(5x + 2 + y)$ **e** $y(9 - y - x)$ **f** $3y(y - 3 + 2x)$

5 $5(2x + 6) + 2(3x - 5) = 10x + 30 + 6x - 10 = 16x + 20 = 4(4x + 5)$

6 $6(3y + 2) - 4(y - 2) = 18y + 12 - 4y + 8 = 14y + 20 = 2(7y + 10) \neq 2(7y + 2)$
The mistake he has made is when he has expanded. He has done $-4 \times -2 = -8$ and so his expansion is $18y + 12 - 4y - 8 = 14y + 4 = 2(7y + 2)$.

Exercise 9.6 Adding and subtracting algebraic fractions

1 **a** $\frac{2x}{5}$ **b** $\frac{4x}{7}$ **c** $\frac{x}{4}$ **d** $\frac{x}{3}$ **e** $\frac{2x}{5}$ **f** $\frac{2x}{3}$
g $\frac{3y}{4}$ **h** $\frac{7y}{9}$ **i** $\frac{7y}{10}$ **j** $\frac{3y}{8}$ **k** $\frac{9y}{25}$ **l** $\frac{3y}{14}$

2 **a** $\frac{a}{2} + \frac{a}{5} = \frac{5a}{10} + \frac{2a}{10} = \frac{5a + 2a}{10} = \frac{7a}{10}$

b $\frac{b}{4} + \frac{b}{3} = \frac{3b}{12} + \frac{4b}{12} = \frac{3b + 4b}{12} = \frac{7b}{12}$

c $\frac{5c}{7} - \frac{2c}{5} = \frac{25c}{35} - \frac{14c}{35} = \frac{25c - 14c}{35} = \frac{11c}{35}$

d $\frac{5d}{6} + \frac{3d}{5} = \frac{25d}{30} + \frac{18d}{30} = \frac{25d + 18d}{30} = \frac{43d}{30}$

e $\frac{5e}{8} + \frac{2e}{3} = \frac{15e}{24} + \frac{16e}{24} = \frac{15e + 16e}{24} = \frac{31e}{24}$

f $\frac{9f}{10} + \frac{3f}{4} = \frac{18f}{20} + \frac{15f}{20} = \frac{18f + 15f}{20} = \frac{33f}{20}$

3 **a** $\frac{x + y}{5}$ **b** $\frac{3x + y}{6}$ **c** $\frac{6x + y}{9}$ **d** $\frac{4x - y}{10}$ **e** $\frac{11x - 4y}{14}$ **f** $\frac{9x - 8y}{20}$
g $\frac{3a + 4b}{12}$ **h** $\frac{12a + 5b}{30}$ **i** $\frac{10a + 9b}{24}$ **j** $\frac{8a - 5b}{40}$ **k** $\frac{9a - 2b}{30}$ **l** $\frac{20a - 27b}{45}$

4 **a** A, D, F **b** B, C, E **c** G; the answer is $\frac{x}{3}$
d You can ignore the letter, work out the fractions, then put the letter back in at the end.

Exercise 9.7 Expanding the product of two linear expressions

1 **a** $(x+4)(x+1) = x^2 + 1x + 4x + 4$
$= x^2 + 5x + 4$

b $(x-3)(x+6) = x^2 + 6x - 3x - 18$
$= x^2 + 3x - 18$

c $(x+2)(x-8) = x^2 - 8x + 2x - 16$
$= x^2 - 6x - 16$

d $(x-4)(x-1) = x^2 - x - 4x + 4$
$= x^2 - 5x + 4$

2 **a** $x^2 + 10x + 21$ **b** $x^2 + 11x + 10$ **c** $x^2 + 2x - 15$
d $x^2 + 4x - 32$ **e** $x^2 - 9x + 14$ **f** $x^2 - 14x + 24$

3 **a** $y^2 + 6y + 8$ **b** $z^2 + 14z + 48$ **c** $m^2 + m - 12$
d $a^2 - 7a - 18$ **e** $p^2 - 11p + 30$ **f** $n^2 - 30n + 200$

4 **a** C **b** B **c** A **d** C

5 **a** $(x+2)^2 = (x+2)(x+2)$
$= x^2 + 2x + 2x + 4$
$= x^2 + 4x + 4$

b $(x-3)^2 = (x-3)(x-3)$
$= x^2 - 3x - 3x + 9$
$= x^2 - 6x + 9$

6 **a** $y^2 + 10y + 25$ **b** $z^2 + 2z + 1$ **c** $m^2 + 16m + 64$
d $a^2 - 4a + 4$ **e** $p^2 - 8p + 16$ **f** $n^2 - 18n + 81$

7 **a** **i** $x^2 - 4$ **ii** $x^2 - 25$ **iii** $x^2 - 49$
b There is no term in x, and the number term is a square number.
c $x^2 - 100$
d $x^2 - y^2$

8 **a** ① $33 \times 29 = 957$, ② $28 \times 34 = 952$, ③ $957 - 952 = 5$
b ① $16 \times 12 = 192$, ② $11 \times 17 = 187$, ③ $192 - 187 = 5$
c The answer is always 5.
d

n	$n+1$
$n+5$	$n+6$

e ① $(n+5)(n+1) = n^2 + 6n + 5$, ② $n(n+6) = n^2 + 6n$, ③ $n^2 + 6n + 5 - (n^2 + 6n) = 5$
The answer is always 5.

End-of-unit review

1 **a** x^5 **b** y^{12} **c** z^{10} **d** $15m^9$ **e** $6n^{11}$ **f** $6p^7$
g q^6 **h** r^3 **i** t^5 **j** $2u^2$ **k** $3v^6$ **l** $7w$

2 **a** $3a$ **b** $2b + 16$ **c** $5c + 2d$ **d** $16z - 2$

3 **a** ab **b** $40cd$ **c** w^2 **d** $9e^2$

4 **a** 13 **b** 19 **c** 13 **d** 54
e 3 **f** 48 **g** −8 **h** 21
i 89 **j** 0 **k** 84 **l** −42

5 **a** $x = 19$ **b** $x = -4$ **c** $y = 65$ **d** $y = 60$ **e** $z = 2$ **f** $z = 6$

6 **a** $2(x+3)$ **b** $4(y-3)$ **c** $3(a-1)$ **d** $10(2-x)$ **e** $6(4+5z)$ **f** $10(5-3b)$
g $x(5x+1)$ **h** $a(3-5a)$ **i** $8(4y-x)$ **j** $3y(2x-1)$ **k** $2m(9+4n)$ **l** $3n(8-9n)$

7 **a** $\frac{2x}{3}$ **b** $\frac{3x}{5}$ **c** $\frac{3x}{7}$ **d** $\frac{2y}{15}$ **e** $\frac{15y}{8}$ **f** $\frac{y}{6}$
g $\frac{x+y}{4}$ **h** $\frac{12x-y}{20}$ **i** $\frac{5a+3b}{15}$ **j** $\frac{15a+8b}{20}$ **k** $\frac{20a-3b}{24}$ **l** $\frac{12a-14b}{21}$

8 **a** $x^2 + 7x + 10$ **b** $x^2 + x - 12$ **c** $x^2 - 3x - 54$
d $x^2 - 14x + 40$ **e** $x^2 - 64$ **f** $x^2 - 12x + 36$

9 $4(2x+5) + 3(8x-4) = 8x + 20 + 24x - 12 = 32x + 8 = 8(4x+1)$